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Decoherence from Isocurvature perturbations in Inflation

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We present a quantitative study of the quantum decoherence of curvature perturbations during inflation in the presence of isocurvature modes. If the latter cannot be observed directly, tracing them out effectively decoheres the curvature perturbation even in the absence of a direct coupling between the scalar fields involved. We then calculate the entanglement entropy and argue that it provides a quantitative measure for decoherence.

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Introduction – Inflation has become the leading paradigm of early universe cosmology not least because of its ability to imprint scale invariant inhomogeneities on super-horizon scales via a causal mechanism. These inhomogeneities are thought to provide the seeds which later become the temperature anisotropies in the Cosmic Microwave Background and the Large Scale Structure in the Universe. The origin of these initial perturbations lies in quantum fluctuations of matter fields, amplified during a period of inflationary quasi-exponential expansion, and interpreted as adiabatic and isocurvature cosmological perturbations. Adiabatic perturbations are variations in total energy density or equivalently in gravitational potentials, while isocurvature perturbations are variations in the entropy density of various matter components. All inflationary models share this basic mechanism, with different models characterized by small deviations from exact scale invariance. Along with other calculable properties, these deviations are particularly appealing as they allow for inflationary models to be tested against increasingly precise observations [1].

Inflationary calculations are based on the quantum mechanics of scalar fields in expanding spacetimes [2], where the relevant observable is the amplitude of the field's Fourier modes. Although treated as a quantum mechanical variable, this amplitude is interpreted as a stochastic random variable described by a gaussian distribution, with the variance given by the power-spectrum. This interpretation, used in CMB analyses and simulations of Large Scale Structure, proves to be very accurate for calculational purposes; quantum correlation functions, exactly calculated, differ very little from the corresponding correlation functions obtained by using the relevant classical probability distribution [3].

The consistency of this stochastic interpretation requires a density matrix which is diagonal in the amplitude basis. However, the density matrix of inflationary perturbations is *not* automatically diagonal in this basis. One cannot assign a specific amplitude to the Fourier modes which must thus be considered to exist in coherent superpositions of different amplitudes. Prior to a stochastic interpretation, an as-of-yet undisclosed decoherence mechanism is required to diagonalize the density matrix.

The phenomenon of decoherence originates from couplings of the system of interest with degrees of freedom belonging to some unobservable environment [4, 5]. While arguments suggesting that a form of such environmental decoherence can indeed occur for inflationary perturbations have been put forward [6, 7, 8, 9], the necessary coupling to some "environment" is either assumed or estimated rather than derived from first principles. Here, we address this issue in a precisely defined dynamical setting where the role of the environment is played by unobservable isocurvature perturbations. Even though the inflationary amplification mechanism is operative on super-Hubble scales, causality does not prevent decoherence from occurring efficiently.

Quantum mechanics of inflationary perturbations– To study decoherence in inflation it is convenient to use the Schrödinger picture. The action for a free massive scalar field is

$$S = -\int d^4x\sqrt{-g}\,\frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi + m^2\phi^2\right) \qquad (1)$$

where we take the spacetime metric to be

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j\,. \qquad (2)$$

Then, the canonical momentum conjugate to the field ϕ,

$$\pi = \frac{\partial\mathcal{L}}{\partial(\partial_t\phi)} = a^3\partial_t\phi \qquad (3)$$

allows us to write the Hamiltonian $H \equiv \int d^3x\,(\pi\partial_t\phi - \mathcal{L})$

$$H = \int d^3x\,\frac{1}{2}\left(\frac{\pi^2}{a^3} + a(\partial_i\phi)^2 + a^3m^2\phi^2\right)\,. \qquad (4)$$

Quantization dictates the replacement $\pi(\mathbf{x}) \to -i\hbar\frac{\delta}{\delta\phi(\mathbf{x})}$ which realizes the commutation relation $[\phi(\mathbf{x}),\pi(\mathbf{y})] = i\hbar\,\delta(\mathbf{x}-\mathbf{y})$. The wave functional $\Psi[\phi]$ obeys the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Psi = \hat{H}\Psi\,. \qquad (5)$$

Since we will be considering perturbations in their ground states, we take the wavefunctional to have a gaussian form

$$\Psi[\phi] = \mathcal{N}\exp\left[-\frac{1}{2}\int d^3x d^3y\,\phi(\mathbf{x})A(\mathbf{x},\mathbf{y},t)\phi(\mathbf{y})\right] \qquad (6)$$

with $\mathcal{N}$ the normalization factor. Due to the homogeneity of the vacuum state, the kernel in (6) satisfies, $A(\mathbf{x},\mathbf{y},t) = A(\mathbf{y},\mathbf{x},t)$. For this state the Schrödinger equation gives

$$\begin{aligned} i\hbar\,\partial_t \ln\mathcal{N} &= \frac{\hbar^2}{2a^3}\int d^3x\, A(\mathbf{x},\mathbf{x},t) \qquad (7)\\ i\hbar\partial_t A(\mathbf{y},\mathbf{z},t) &= -\frac{\hbar^2}{2a^3}\int d^3x A(\mathbf{x},\mathbf{y},t)A(\mathbf{x},\mathbf{z},t)\\ &- \frac{a}{2}\left(\partial^2_{\mathbf{y}} - a^2m^2\right)\delta^{(3)}(\mathbf{y}-\mathbf{z})\,. \qquad (8)\end{aligned}$$

Since the theory is noninteracting, it is convenient to solve (8) in momentum space, where it becomes local. Indeed, upon writing

$$\begin{aligned} \phi(\mathbf{x}) &= \int \frac{d^3k}{(2\pi)^3}\,\phi_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{x}}\,, \qquad (9)\\ A(\mathbf{x},\mathbf{y},t) &= \int \frac{d^3k}{(2\pi)^3}\,A(\mathbf{k},t)e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\,, \qquad (10)\end{aligned}$$

with $\phi_{-\mathbf{k}} = \phi_{\mathbf{k}}{}^{\star}$ and $A(-\mathbf{k},t) = A(\mathbf{k},t)$, we get

$$i\hbar\,\partial_t A(\mathbf{k},t) = -\frac{\hbar^2}{2a^3}A^2(\mathbf{k},t) + \frac{a}{2}(k^2 + a^2m^2)\,. \qquad (11)$$

The *Heisenberg picture* mode functions $\phi_{\mathbf{k}}(\eta)$ of a scalar field with the action (1) satisfy

$$\partial^2_\eta(a\phi_{\mathbf{k}}(\eta)) + \left(k^2 + \frac{\frac{1}{4}-\nu^2}{\eta^2}\right)(a\phi_{\mathbf{k}}(\eta)) = 0\,, \qquad (12)$$

where η is the conformal time, defined via $ad\eta = dt$, and $\nu^2 = \frac{9}{4} - \frac{m^2}{H^2}$. The Bunch-Davies vacuum solution of (12) is of the form,

$$\phi_{\mathbf{k}}(\eta) = \sqrt{-\frac{\pi\eta}{4}}e^{i\frac{\pi}{2}\left(\nu+\frac{1}{2}\right)}H^{(1)}_\nu(-k\eta)\,, \qquad (13)$$

and the kernel $A(\mathbf{k})$ (11) is related to the mode functions (13) via [2]

$$A(\mathbf{k},\eta) = \frac{1}{2\hbar|\phi_{\mathbf{k}}(\eta)|^2}\left(1 - i\,a^2\partial_\eta|\phi_{\mathbf{k}}(\eta)|^2\right)\,. \qquad (14)$$

At early times, in the regime $k^2\eta^2 \gg 1$ when the mode is "deep inside the horizon" and spacetime curvature is not important, the solution (13) or equivalently (6) and (14) reduces to that of Minkowski vacuum. At late times, when $k^2\eta^2 \ll 1$, the mode has "exited the horizon" and behaves like a time-dependent inverted harmonic oscillator, resulting in amplification. The wave function for each mode can be written as[1]

$$\Psi(\phi_{\mathbf{k}},\eta) \propto \exp\left(-\frac{1}{2}\,\phi_{\mathbf{k}}A(\mathbf{k},\eta)\phi^{\star}_{\mathbf{k}}\right)\,, \qquad (15)$$

where the normalisation factor can be easily obtained from (7) and by requiring $\int d\phi d\phi^{\star}|\Psi|^2 = 1$. From now on we shall be ignoring such factors. Note that in (15) $\phi_{\mathbf{k}}$ is a *time independent* configuration variable in the Schrödinger picture and should not be confused with the Heisenberg picture mode $\phi_{\mathbf{k}}(\eta)$ (13). We have explicitly written out the time dependence of the latter to indicate this difference. In the Schrödinger picture the time evolution of the wave function is determined by the kernel $A(\mathbf{k},\eta)$.

The quantum mechanics of the inverted harmonic oscillator has been studied in [2, 10]. The system evolves into a kind of state known as a squeezed state, which exhibits a high degree of WKB classicality, $\hat{p}\,\Psi \simeq \partial_q S(q)\Psi$, where $\hat{q}$ and $\hat{p}$ are the configuration variable and its conjugate momentum, while S is the exponent of the wave function, practically the classical action. Thus, the amplitude of each mode and its conjugate momentum are related to a very high degree of accuracy via the corresponding classical relation. In cosmology, this state of affairs is interpreted as equivalent to a statistical mixture of mode amplitude eigenstates, where the probability that any of these states has been realized in our universe is given by $|\Psi(\phi_{\mathbf{k}},\eta)|^2$. This would correspond to a density matrix which is diagonal in the field amplitude basis

$$\hat{\rho}_{\mathbf{k}} = \sum_{\phi_{\mathbf{k}}}|\Psi(\phi_{\mathbf{k}},\eta)|^2\,|\phi_{\mathbf{k}}\rangle\langle\phi_{\mathbf{k}}|. \qquad (16)$$

Such a stochastic mixture is a very good approximation when calculating correlators such as $\langle\phi^2_{\mathbf{k}}\rangle$ [3], which are then related to the stochastic properties of cosmological perturbations. However, the wave function (15) does not directly lead to the interpretation suggested by (16). Explicitly calculating the density matrix from (15), we find

$$\begin{aligned} &\rho(\phi_{\mathbf{k}},\bar{\phi}_{\mathbf{k}}) \equiv \Psi(\phi_{\mathbf{k}},\eta)\Psi^{\star}(\bar{\phi}_{\mathbf{k}},\eta) \qquad (17)\\ &\propto \exp\left(-u_{\mathbf{k}}\Re[A]u^{\star}_{\mathbf{k}} - \tfrac{1}{4}\Delta_{\mathbf{k}}\Re[A]\Delta^{\star}_{\mathbf{k}} - i\Im[A]\Re[u_{\mathbf{k}}\Delta^{\star}_{\mathbf{k}}]\right)\,,\end{aligned}$$

where we have defined $u = (\phi+\bar{\phi})/2$ and $\Delta = \phi-\bar{\phi}$. The above expression does not vanish for $\Delta \neq 0$, making the off-diagonal terms of the density matrix non-zero. Thus, strictly speaking, the description (16) is not valid and one cannot say that any particular mode amplitude has been realized with a certain probability. The modes of cosmological perturbations exist in coherent superpositions with different mode amplitudes. In order to diagonalize the density matrix, some process of decoherence must take place. As we show below, the existence of fields other than the inflaton during inflation can decohere the density matrix of cosmological perturbations even if no direct coupling between the different fields is assumed.

Isocurvature perturbations and decoherence – Consider a two-field model of inflation

$$V(\phi,\chi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}\mu^2\chi^2\,, \qquad (18)$$

[1] The reality of the field $\phi(\mathbf{x})$ implies that the modes $\phi_{-\mathbf{k}}$ and $\phi_{\mathbf{k}}$ are related as $\phi_{-\mathbf{k}} = \phi^{\star}_{\mathbf{k}}$, which necessitates that wave functions be represented in terms of two-mode states [10].

with $\mu^2/m^2 \gg 1$. We take inflation to be dominated by the light field, $m^2\phi^2 > \mu^2\chi^2$, and assume the fields to be in slow-roll,

$$\dot{\phi} \simeq -\sqrt{\frac{2}{3}} M_p m\,, \quad \dot{\chi} \simeq -\frac{\mu^2}{3H}\chi\,, \tag{19}$$

with $H^2 = (\dot{a}/a)^2 \simeq m^2\phi^2/(6M_p^2)$, $M_p = (8\pi G)^{-1/2}$, giving for χ and a

$$\chi(t) \simeq \chi(t_0) e^{-\frac{\mu^2}{3H}t}\,, \quad a(t) \simeq \exp(Ht)\,. \tag{20}$$

After a sufficient amount of time the heavy field decays to its valley $\chi \to 0$, while $\dot{\chi} \ll \dot{\phi}$. The validity of slow roll requires $\epsilon \ll 1$, where

$$\epsilon = \frac{M_p^2}{2}\left(\frac{\partial_\phi V}{V}\right)^2 = 2\frac{M_p^2}{\phi^2} \quad \Rightarrow \quad \frac{m^2}{H^2} = 3\epsilon\,. \tag{21}$$

The condition $\eta_\alpha = M_p^2 \partial_\alpha^2 V/V < 1\,(\alpha = \{\phi, \chi\})$ is also required, which dictates

$$\frac{\mu^2}{m^2}\epsilon \; = \frac{1}{3}\frac{\mu^2}{H^2} < 1\,. \tag{22}$$

We consider the case $\mu^2/m^2 \gg 1$ while still preserving $\mu^2/H^2 < 1$. Perturbing both the metric and the scalar fields

$$\begin{aligned} &\phi(\mathbf{x},t) = \phi(t) + \delta\phi(\mathbf{x},t), \quad \chi(\mathbf{x},t) = \chi(t) + \delta\chi(\mathbf{x},t) \\ &ds^2 = -(1+2\Phi(\mathbf{x},t))dt^2 + a^2(t)(1-2\Phi(\mathbf{x},t))d\mathbf{x}^2\,, \end{aligned}$$

we can form the gauge invariant variables $q_\phi = a(\delta\phi + \frac{\Phi}{H}\dot{\phi})$ and likewise for χ. Then, to leading order in slow roll the equations governing cosmological perturbations are

$$\partial_\eta^2 q_\alpha + \left(k^2 + \frac{\frac{1}{4} - \nu_\alpha^2}{\eta^2}\right) q_\alpha = 0\,, \tag{23}$$

where

$$\nu_\phi^2 \simeq \frac{9}{4} + 2\epsilon\,, \quad \nu_\chi^2 \simeq \frac{9}{4} - \frac{\mu^2}{m^2}3\epsilon - \epsilon\,. \tag{24}$$

Thus, in the above model cosmological perturbations can be approximated by two decoupled scalar fields with different masses. In the long wavelength limit we have

$$\lim_{k \ll aH} q_{\alpha k} \simeq \frac{e^{i\frac{\pi}{2}\left(\nu_\alpha - \frac{1}{2}\right)}}{\sqrt{2k\pi}} \Gamma(\nu_\alpha)\left(-\frac{k\eta}{2}\right)^{-\nu_\alpha + \frac{1}{2}}\,. \tag{25}$$

Using this and (14), we find to leading order for the kernel $A(\mathbf{k}, \eta)$

$$A_\alpha = \frac{a^3 H}{\hbar}\left(\frac{2\pi}{\Gamma(\nu_\alpha)^2}\left(\frac{k}{2aH}\right)^{2\nu_\alpha} + i\left(\frac{3}{2} - \nu_\alpha\right)\right)\,, \tag{26}$$

and the wave function of the system is written as

$$\Psi(q_\phi, q_\chi) = \exp\left[-\frac{1}{2}(q_\phi, q_\chi)\mathbf{A}\begin{pmatrix} q_\phi^\star \\ q_\chi^\star \end{pmatrix}\right]\,, \tag{27}$$

where $\mathbf{A} \simeq \mathrm{diag}(A_\phi, A_\chi)$.

The above model produces two types of perturbations completely decoupled within our approximations. However, the perturbations q_ϕ and q_χ are not directly observable. Cosmological large scale structure is determined by the adiabatic curvature perturbation or equivalently the Newtonian potential Φ. From the $0i$ Einstein equation we find

$$\dot{\Phi} + H(1 - \dot{H}/H^2)\Phi = \frac{1}{2M_p^2}\left(\dot{\phi}\,\frac{q_\phi}{a} + \dot{\chi}\,\frac{q_\chi}{a}\right)\,, \tag{28}$$

so that the gravitational potential is related to the part of q_α parallel to the field velocity in field space. The components normal to the trajectory are *isocurvature* perturbations.

To a first approximation, Φ is determined by q_ϕ while $q_\chi \simeq a\delta\chi$ is the isocurvature perturbation. After the heavier field χ has entered its valley, inflation proceeds in the direction of ϕ, becoming effectively single field; the perturbations related to the χ direction contribute to the energy density only at second order in the perturbations. Unless some curvaton type mechanism [11, 12] makes them dominant later, they are unobservable as they have no effect on the curvature perturbation.

The above picture is a very good approximation for calculating the power spectrum. However, if χ is not placed exactly at $\chi = 0$, its evolution can have a dramatic effect on the quantum character of the perturbations. In reality, the perturbations in ϕ and χ need to be rotated to yield the adiabatic (q_σ) and isocurvature (q_s) components [13, 14]

$$\begin{pmatrix} q_\sigma \\ q_s \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \delta\phi \\ \delta\chi \end{pmatrix}\,, \tag{29}$$

where

$$\cos\theta = \frac{\partial_t\phi}{\sqrt{(\partial_t\phi)^2 + (\partial_t\chi)^2}}\,, \quad \sin\theta = \frac{\partial_t\chi}{\sqrt{(\partial_t\phi)^2 + (\partial_t\chi)^2}}\,. \tag{30}$$

The equation for the gravitational potential is now expressed as

$$\dot{\Phi} + H(1 - \dot{H}/H^2)\Phi = \frac{1}{2M_p^2}\sqrt{\dot{\phi}^2 + \dot{\chi}^2}\,\frac{q_\sigma}{a}\,, \tag{31}$$

making q_σ the relevant variable for cosmological observations since it sources the gravitational potential. Expressing the wave function in terms of q_σ and q_s, we have

$$\Psi(q_\sigma, q_s) = \exp\left[-\frac{1}{2}(q_\sigma, q_s)\,\mathbf{C}\begin{pmatrix} q_\sigma^\star \\ q_s^\star \end{pmatrix}\right]\,, \tag{32}$$

where

$$\mathbf{C} \equiv \begin{pmatrix} \frac{A_\phi + A_\chi}{2} + \frac{A_\phi - A_\chi}{2}\cos 2\theta & -\frac{A_\phi - A_\chi}{2}\sin 2\theta \\ -\frac{A_\phi - A_\chi}{2}\sin 2\theta & \frac{A_\phi + A_\chi}{2} - \frac{A_\phi - A_\chi}{2}\cos 2\theta \end{pmatrix}\,. \tag{33}$$

The reduced density matrix is now constructed by tracing out the isocurvature perturbations q_s, which we assume to be irrelevant for observations

$$\tilde{\rho}(q_\sigma,\bar{q}_\sigma) = \int dq_s\, dq_s^\star \Psi(q_\sigma,q_s)\Psi^\star(\bar{q}_\sigma,\bar{q}_s)\,. \tag{34}$$

Expressing the result in terms of the variables $u = (q_\sigma + \bar{q}_\sigma)/2$ and $\Delta = q_\sigma - \bar{q}_\sigma$, we find

$$\tilde{\rho}(u,\Delta) = \exp\left[-\frac{1}{2}(u,\Delta)\mathbf{D}\begin{pmatrix}u^\star\\ \Delta^\star\end{pmatrix}\right] \tag{35}$$

with

$$\mathbf{D} \equiv \begin{pmatrix} 2\left(\Re[C_{11}] - \frac{\Re[C_{12}]^2}{\Re[C_{22}]}\right) & i\left(\Im[C_{11}] - \frac{\Re[C_{12}]\Im[C_{12}]}{2\Re[C_{22}]}\right) \\ i\left(\Im[C_{11}] - \frac{\Re[C_{12}]\Im[C_{12}]}{2\Re[C_{22}]}\right) & \frac{1}{2}\left(\Re[C_{11}] + \frac{\Im[C_{12}]^2}{\Re[C_{22}]}\right)\end{pmatrix}. \tag{36}$$

As we remarked above, decoherence implies that the coefficient of the Δ components is large enough so that they are suppressed when $\Delta > 0$. If $D_{11}/D_{22} \to 0$, the relative importance of the Δ over the u terms will diminish, making quantum correlations unimportant. From (33) and (36) we find

$$\frac{D_{11}}{4D_{22}} = \left(1 + \frac{|A_\phi - A_\chi|^2}{4\Re[A_\phi]\Re[A_\chi]}\sin^2(2\theta)\right)^{-1}, \tag{37}$$

and by using (23), (24), (26) and (27) we have

$$\frac{|A_\phi - A_\chi|^2}{4\Re[A_\phi]\Re[A_\chi]}\sin^2(2\theta) \simeq \frac{\Gamma^2(\nu_\phi)\Gamma^2(\nu_\chi)}{8\pi^2}\left(\frac{\mu^2}{m^2}\right)^4\epsilon^3\left(\frac{\chi_0}{M_p}\right)^2 \times\left(\frac{2aH}{k}\right)^{6-2\epsilon\mu^2/m^2} a^{-2\epsilon\mu^2/m^2}, \tag{38}$$

demonstrating that decoherence proceeds very fast. Indeed, $D_{11}/D_{22} \propto a^{-6+4\epsilon\mu^2/m^2}$ such that eventually $D_{22} \gg D_{11}$. Alternatively, the decoherence rate for the reduced density matrix (35) scales as $\Gamma_\Delta \propto a^{6-4\epsilon\mu^2/m^2}$, where $\dot{\tilde{\rho}} = -\Gamma_\Delta\Delta\Delta^*\tilde{\rho} + \ldots$ Although we have only treated the case $\mu^2/m^2 \gg 1$ for simplicity, decoherence of adiabatic by isocurvature perturbations persists in the regime $0 < \mu^2 - m^2 \ll \epsilon H^2$ [15].

Entropy – The analysis in [16] implies that the entropy of a gaussian quantum state can be expressed in terms of the product of the field amplitude and momentum variances (Δ_ϕ^2, Δ_π^2) as

$$S \;=\; \mathrm{tr}\ln\left[2\Delta_\phi\Delta_\pi/\hbar\right], \tag{39}$$

with $S = 0$ for any minimum uncertainty state. Note that before decoherence $\Delta_\phi^2 = 1/\Re[A_\mathbf{k}]$ and $\Delta_\pi^2 = \hbar^2\Re[A_\mathbf{k}]/4$ such that the state (15) corresponds to the minimum uncertainty state with vanishing entropy. Calculating the entropy corresponding to the reduced density matrix (35), we get for the decohered adiabatic perturbations

$$S = V\int\frac{d^3k}{(2\pi)^3}\, s_\mathbf{k}\,, \tag{40}$$

with

$$s_\mathbf{k} = \frac{1}{2}\ln\left[1 + \frac{4D_{22}}{D_{11}}\right] \simeq \left(3 - 2\frac{\mu^2}{m^2}\epsilon\right)\ln\left(\frac{aH}{k}\right) + s_{\mathbf{k}=Ha}\,, \tag{41}$$

where $s_\mathbf{k} = s_\mathbf{k}(\eta)$ denotes the entropy per mode pair $\{\mathbf{k}, -\mathbf{k}\}$ and $s_{\mathbf{k}=Ha}$ is the entropy per mode at the Hubble crossing. From $s_\mathbf{k} \simeq \ln[2n_\mathbf{k}]$, one can define an effective particle number per mode [*cf.* (37),(38)] $n_\mathbf{k} \simeq 2\Delta_\phi\Delta_\pi/\hbar \simeq 2\sqrt{D_{22}/D_{11}}$, generated by the dynamical decoherence process considered here. This should not be confused with the particle number $n_\mathbf{k} \simeq (2aH/k)^2$ [16, 17], usually associated with the Bunch-Davies vacuum (which is a pure state). Indeed, one can show [15] that $s_\mathbf{k} \to 0$ in the limit when $\mu \to m$, in which case isocurvature and adiabatic modes decouple and decoherence ceases. Equation (41) establishes a link between entropy and decoherence by defining a precise relation between them; the entropy (41) corresponds to the entanglement entropy which equals (minus) the information stored in correlations between the adiabatic and isocurvature perturbations. Therefore, it conforms with the standard definition of the entropy of quantum systems: tracing out the isocurvature perturbations (the unobservable "environment") generates the entropy of the adiabatic perturbations (the system).

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